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BRIEF COMMUNICATION

THE USE OF FRACTAL TECHNIQUES FOR FLOW REGIME IDENTIFICATION

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INTRODUCTION

The first proposed flow regime maps were based on visual identification of phase distribution. While visual flow regime identification may be adequate for some cases, for many situations these methods are inapplicable or too subjective.

Several other methods have been used previously in an attempt to more objectively classify flow regimes. In particular, Jones & Zuber (1975) and Vince & Lahey (1982) employed transient X-ray attenuation techniques, and calculated the power spectral density (PSD) function and the probability density function (PDF) for chordal void fraction fluctuations. In contrast, Hubbard & Dukler (1966) calculated the PSD for two-phase pressure drop signals, while Tutu (1982, 1984) calculated the PDF of similar signals. Finally, Matsui (1984, 1986) calculated both the PSD and PDF of transient pressure drop signals for various flow regimes.

While these studies have all contributed to our understanding of flow regime classification, currently there is no accepted method to objectively distinguish between flow regimes. It is the purpose of this paper to present a promising new approach using fractal techniques for flow regime identification and classification.

DISCUSSION OF ANALYSIS

In this paper some new measurements of pressure drop fluctuations in a horizontal air/water two-phase flow are discussed. The flow regimes investigated were wavy, plug, slug and annular flows. The PDF and PSD of the signals were calculated, and fractal techniques were also used in an attempt to classify the various flow regimes.

Two-phase flow is a nonlinear dissipative dynamic system which may exhibit a class of motion that is chaotic. The most common manifestation of chaotic systems are the so-called strange attractors, which are asymptotic evolutions of the system's state variables describing fractal trajectories in phase space.

If only one variable related to the flow is available, the state-space trajectories of the motion, or strange attractor, can be reconstructed using a pseudo-phase-space technique (Moon 1987). In a system having n state variables, the attractor normally studied is a subset of an n-dimensional state space.

To explore the possibility of using quantitative measurements of the attractor as a flow regime indicator, several fractal dimensions (Moon 1982) were calculated. Using the digitized time series data which represent the pressure drop fluctuations of a given flow pattern, an n-dimensional state space was reconstructed. The state vector, χ_i, which defines the state space, is given by

$$\chi_i = \{X(i), X(i+\eta), X(i+2\eta), \ldots, X(i+(n-1)\eta)\}^{\mathrm{T}},$$

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where $\{X(1), X(2), \ldots, X(i)\}^{\mathrm{T}}$ represents the matrix vector of the digitized transient pressure drop data and η is a time shift.

To quantify the fractal dimension of the resultant attractor, the so-called correlation dimension, v, proposed by Grassberger & Procaccia (1983), was used. The correlation integral is well approximated by

$$C(r) = \lim_{m \to \infty} \frac{1}{m^2} \sum_{j}^{m} \sum_{\substack{k \\ (k \neq j)}}^{m} \mathscr{H}(r - |\chi_j - \chi_k|), \qquad [1]$$

where $\mathscr{H}$ is the Heaviside function, $|\chi_j - \chi_k|$ is the scalar distance between points on the solution trajectory and r is an arbitrary distance. The correlation integral measures the number of points on the solution trajectories that lie within a hypersphere of radius r which is centered at χ_j. Since many attractors are known to exhibit a power law dependence on r, the correlation dimension, v, for relatively small r, is defined as (Moon 1987):

$$v \triangleq \frac{\mathrm{d}[\log C(r)]}{\mathrm{d}[\log r]}. \qquad [2]$$

To analyze the nature of the chaotic motion related to the flow patterns, the method proposed by Hurst was also used. According to this technique, the so-called rescaled range (R/S), of records in time, is well-described by the following empirical relation (Feder 1988; Mandelbrot 1982):

$$\frac{R}{S} = \left(\frac{\tau}{2}\right)^{H}, \qquad [3]$$

where

$$\bar{\xi} = \frac{1}{n} \sum_{i=1}^{n} \xi_i, \qquad [4a]$$

$$\lambda(n, \tau) = \sum_{i=1}^{n} [\xi_i - \bar{\xi}], \qquad [4b]$$

$$R(\tau) = \max_{n \in \tau} \lambda(n, \tau) - \min_{n \in \tau} \lambda(n, \tau) \qquad [4c]$$

and

$$S = \left[\frac{1}{\tau} \sum_{i=1}^{\tau} (\xi_i - \bar{\xi})^2\right]^{1/2}. \qquad [4d]$$

For many natural phenomena the Hurst dimension, H, which is a type of fractal dimension, has been found to be >0.5, while for independent random processes with finite variance, it is well correlated by $H = 0.5$.

In the equation above, if τ is the number of points of a subset of the time series $\{X(1), X(2), \ldots, X(i)\}^{\mathrm{T}}$, its standard deviation is S, $\lambda(n, \tau)$ is the accumulated departure from the mean and R is the difference between the maximum and minimum value of $\lambda(n, \tau)$.

An algorithm for computing the correlation dimension and plotting the $\log C(r)$ vs $\log r$ curve was developed and applied over both 10,000 and 5000 points of the pressure drop time series taken in this study. No significant improvement was found when the larger number of points was adopted. The Hurst dimension was also calculated to investigate some specific aspects of the processes occurring. The fluctuation pressure drop signal was time-averaged until a statistical stationary process was achieved. The averaged value was then subtracted from the signal. In this way, only the fluctuating component is considered. The PDFs were easily found through the construction of histograms, and a fast-Fourier transform (FFT) algorithm was applied to calculate the PSDs.

DISCUSSION OF THE EXPERIMENT

The test section was mounted horizontally, and comprised a 19 mm i.d. Plexiglas pipe. The development section was 154 L/D long, so that a fully developed flow was achieved in the test section. The test section was 96 L/D long and the mixture was discharged into a separation tank.

Two static pressure taps were mounted flush with the pipe wall near the end of the test section and were axially separated by 8 L/D. They were linked through small tubing and valves to calibrated high-frequency (i.e. 5 kHz natural frequency) Validyne variable differential reluctance transducers and Carrier demodulators to measure the transient pressure drop.

DISCUSSION AND RESULTS

As can be seen in figure 1, an aperiodic pressure drop fluctuation was characteristic of the horizontal flow patterns studied.

The PDF of each flow regime was computed, and shown in figure 2. Multimodal PDFs were found for wavy, annular and slug flow, but the number and location of the peaks was not very distinct. In contrast, unimodal PDFs were found for plug flow. These results are consistent with those of Matsui (1984, 1986), who took two-phase pressure drop data in a vertical pipe. Unfortunately, the shape of the PDFs is not explicit enough to be used as a criterion to distinguish between all flow regimes.

The PSD functions seen in figure 3 show similar features for both separated and intermittent flows. An almost continuous range of frequencies characterize wavy and annular flows, while, as shown in figure 3, a multi-peaked spectra is associated with slug and plug flow. The range of

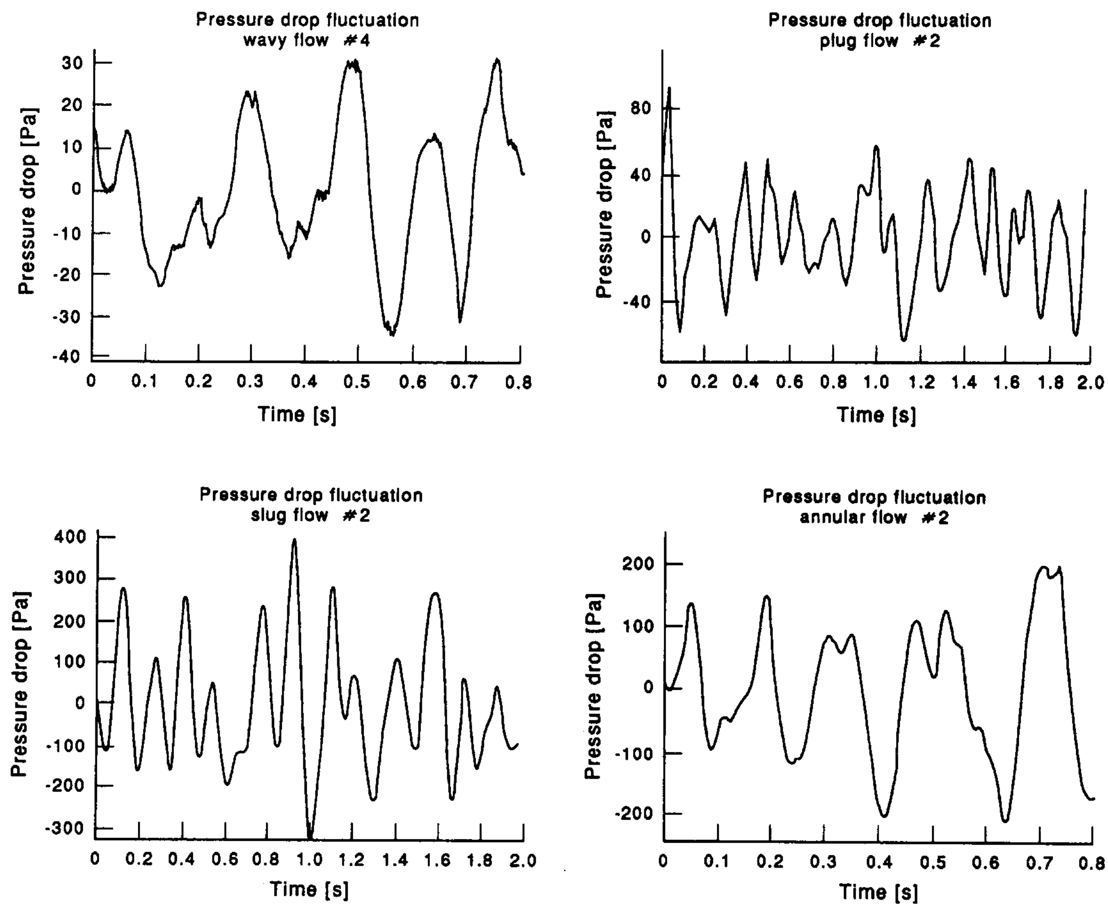

Figure 1. Typical pressure drop signals for various flow regimes.

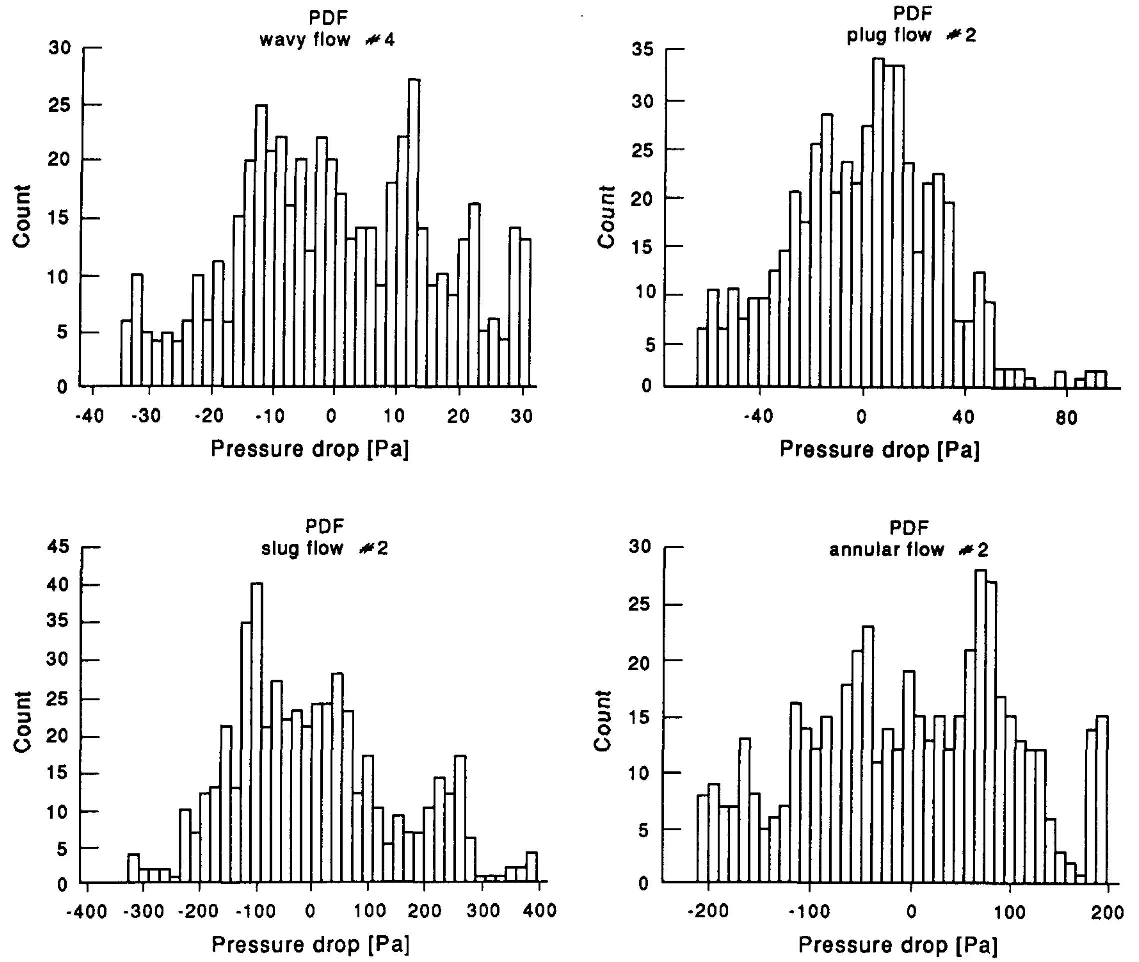

Figure 2. Typical PDFs for various flow regimes.

dominant frequencies, however, is closely related to the volumetric flux of the two-phase mixture, with higher velocities resulting in increasing values of frequency. As a result, the PSD function cannot be easily adopted as a numerical criterion for flow regime identification.

The correlation dimensions plots, for state-space dimensions (n) ranging from 2 to 12, for the flow regimes studied are shown in figure 4. A noisy region, related to very low hypersphere radii, is apparent for all flow regimes. The magnitude of those signals, which are amplified by $(n)^{1/2}$ as the state-space dimensions (n) increases, is unique to each flow pattern.

Beyond the noisy region, a distinct double-sloped curve was found for wavy and annular flow. Multiple slopes in correlation dimension plots have also been found by Ben-Mizrachi *et al.* (1984), who added low-amplitude random noise to deterministic chaos (i.e. the Lorenz attractor). The superimposed nature of the signal was revealed and that part related to the noise (i.e. for smaller hypersphere radii) showed an increasing slope, with no embedding dimension (N). That is, there is no minimum number of deterministic equations which can be used to describe a random process. Separate calculations performed for superimposed signals have also revealed that the relative length of the two slopes is related to the magnitude of each signal.

The correlation dimension plots for wavy and annular flows, had a two-sloped curve, which suggests a superposition of deterministic signals. As has been discussed previously by Hagiwara (1988), the pressure drop signal for annular horizontal flow is apparently made up of the superposition of the signal from the roll waves and surface waves. In our case, that part of the correlation dimension curve related to the surface waves, for both wavy and annular flow, leads to a saturation in the slope (i.e. v) at a lower order embedding dimension than that related to the

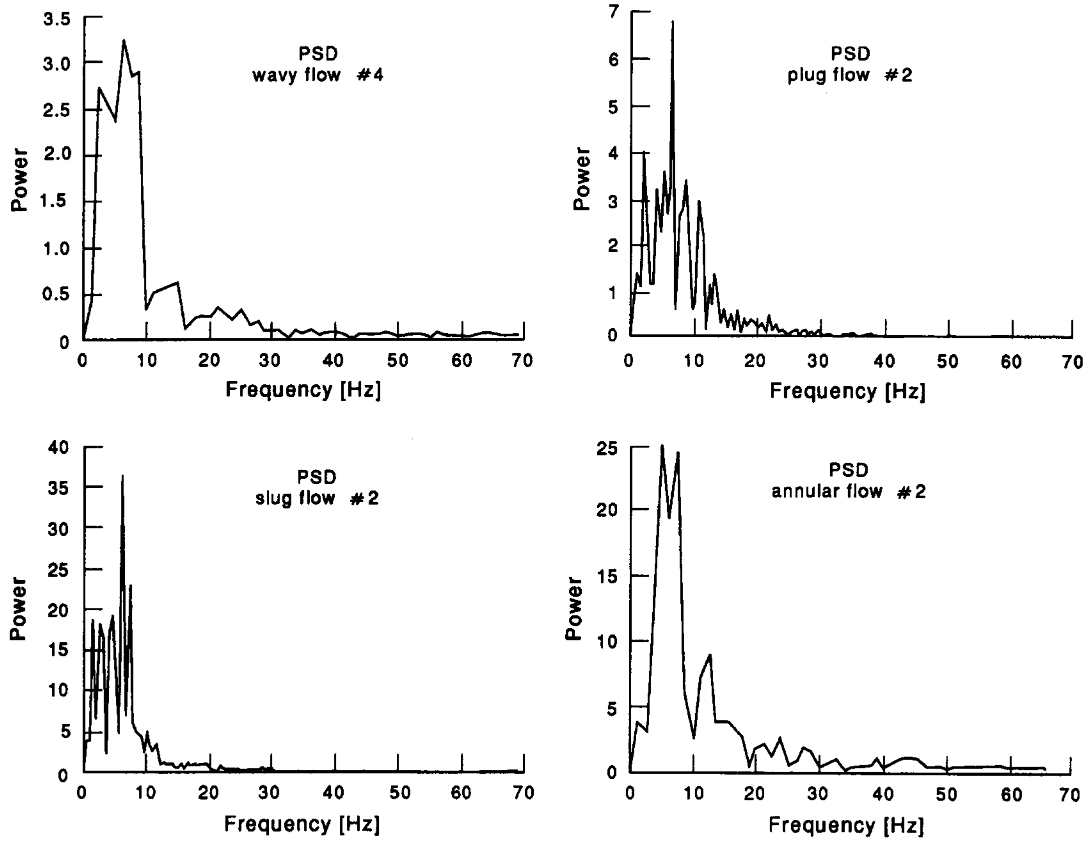

Figure 3. Typical PSDs for various flow regimes.

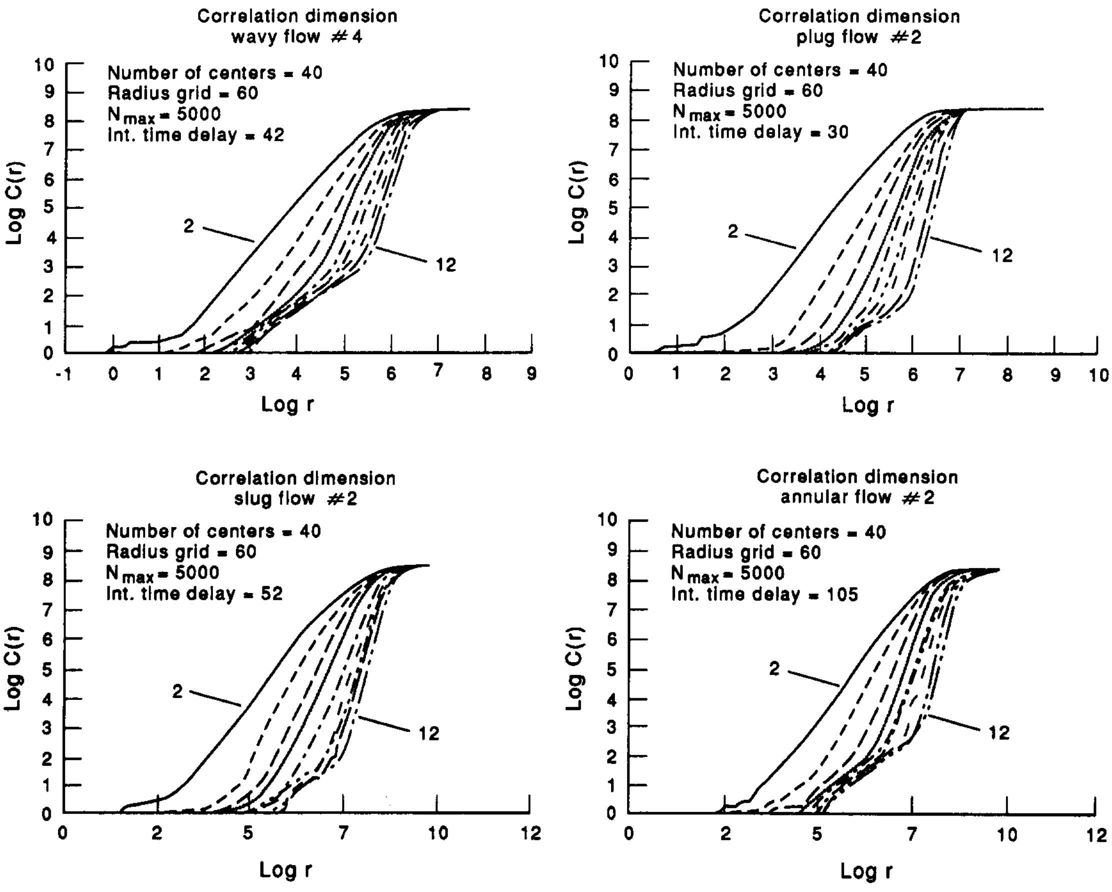

Figure 4. Correlation dimension (ν) plots for various flow regimes.

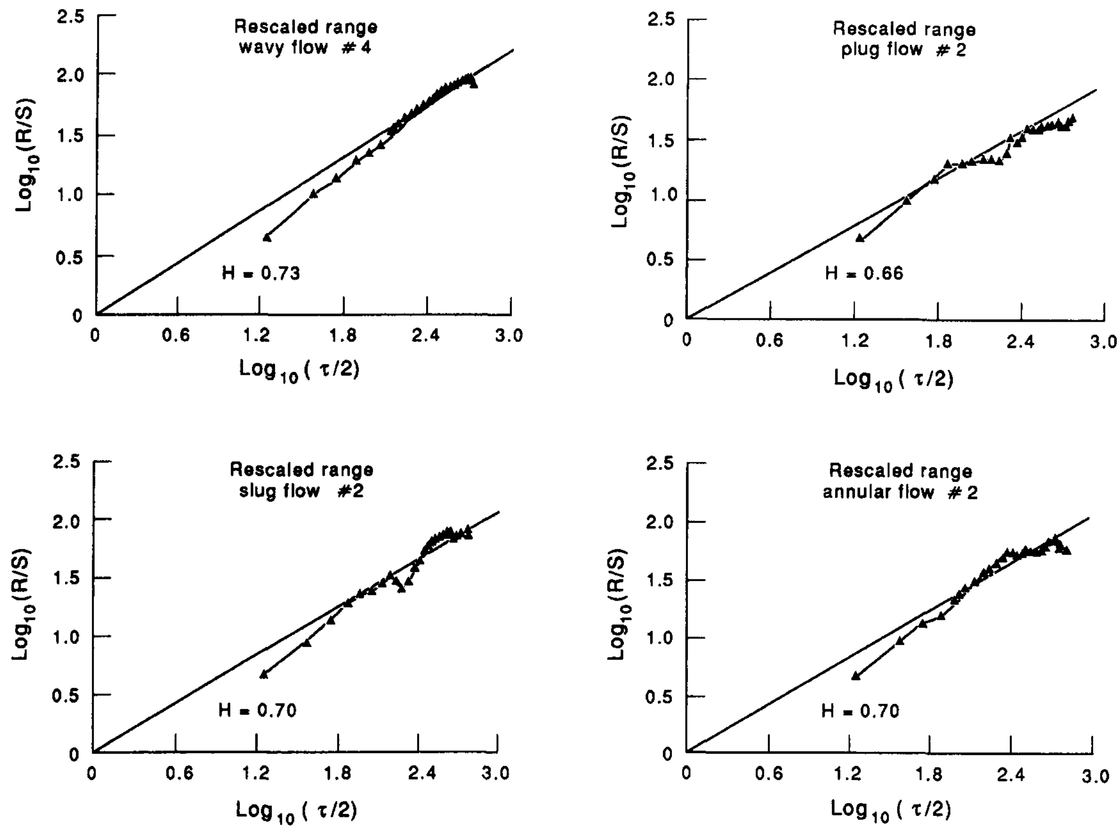

Figure 5. The Hurst dimension (H) of various flow regimes.

Table 1. Fractal dimensions of various flow patterns

Flow	j_G (m/s)	j_L (m/s)	ν	N	H
Wavy	12.60	6.80	1.03/6.21	6/9	0.73
Plug	0.24	1.10	7.17	11	0.66
Slug	0.99	0.82	5.07	9	0.70
Annular	16.45	0.36	0.97/5.93	5/11	0.70

roll waves. In contrast, as can be seen in figure 4, the intermittent plug and slug flows gave rise, beyond the noise level, to single-sloped correlation dimension curves.

The results obtained from a Hurst analysis reinforced the deterministic nature of the signals being analyzed. Figure 5 shows the rescaled range (R/S) plotted against $\tau/2$ for four different flow patterns. The slope of the straight lines, drawn arbitrarily close to the saturation value for the rescaled range curve, indicates the deterministic nature of the flows. The values of the Hurst dimension (H) ranged from 0.66 for plug flow up to 0.73 for wavy flow, thus confirming that the flow regimes studied were chaotic.

Table 1 presents a summary of the correlation dimension (ν), embedding dimension (N) and Hurst dimension (H) for the flow conditions studied.

Finally, figures 6a and 6b present pseudo†-phase-plane plots of the measured pressure drop for wavy flow and slug flow, respectively. It is obvious that the phase-plane signature of these two flow regimes is quite different, and can thus be used to discriminate between these two-phase flow patterns.

It appears that fractal techniques offer a promising way to objectively classify flow patterns. Indeed, while it is difficult to detect structure in the temporal data, the underlying structure related

†Where the time delay, η, was taken as the period associated with the peak in the PSD shown in figure 3.

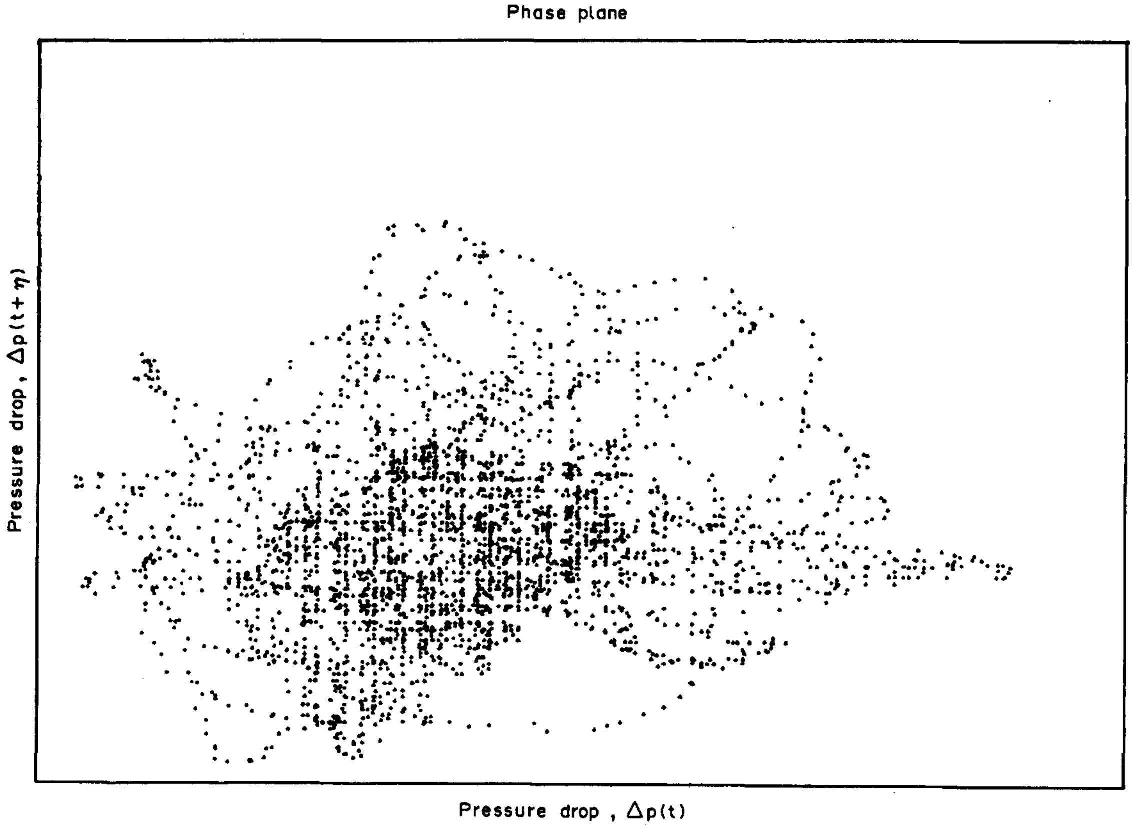

Figure 6a. Pseudo-phase-plane plot for wavy flow.

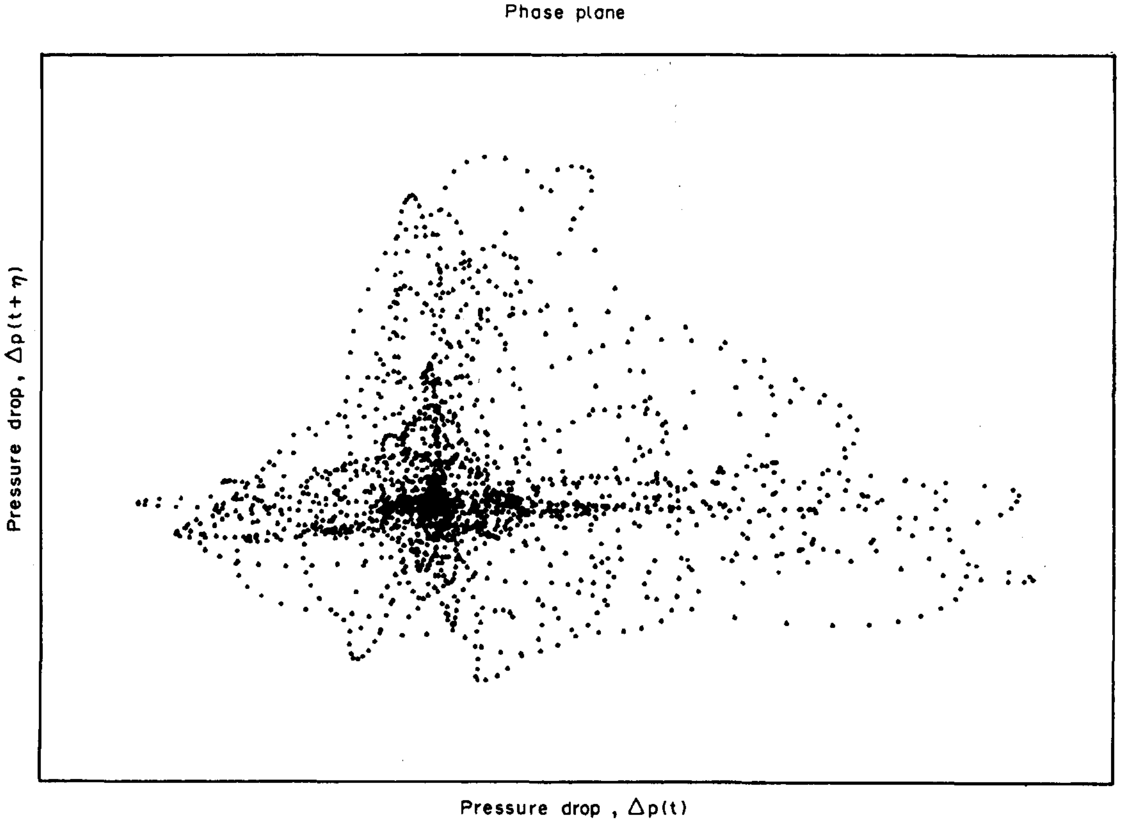

Figure 6b. Pseudo-phase-plane plot for slug flow.

to separated flows was revealed using fractal techniques. Phase-plane plots and differences in the various fractal dimensions appear to be promising for objectively discriminating between separated and intermittent flows. However, more work is needed before truly objective techniques are available for flow regime discrimination. It is hoped that this study will help stimulate the needed research to accomplish this important goal.

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REFERENCES

BEN MIZRACHI, A., PROCACCIA, I. & GRASSBERGER, P. 1984 Characterization of experimental (noisy) strange attractors. *Phys. Rev.* **A29,** 975–977.

FEDER, J. 1988 *Fractals.* Plenum Press, New York.

GRASSBERGER, P. & PROCACCIA, I. 1983 Measuring the strangeness of strange attractors. *Physica* **D,** 189–208.

HAGIWARA, Y. 1988 Experimental studies on chaotic behavior of liquid film flow in annular two-phase flows. *PhysicoChem. Hydrodynam.* **10,** 135–147.

HUBBARD, M. G. & DUKLER, A. E. 1966 The characterization of flow regimes for horizontal two-phase flow. In *Proc. Heat Transfer and Fluid Mechanics Institute.* Stanford Univ. Press, Stanford, Calif.

JONES, O. C. & ZUBER, N. 1975 The interrelation between void fraction fluctuations and flow patterns in two-phase flow. *Int. J. Multiphase Flow* **2,** 273–306.

LIN, P. Y. & HANRATTY, T. J. 1987 Detection of slug flow from pressure measurements. *Int. J. Multiphase Flow* **13,** 13–21.

MANDELBROT, B. B. 1982 *The Fractal Geometry of Nature.* Freeman, New York.

MATSUI, G. 1984 Identification of flow regimes in vertical gas liquid two-phase flow using differential pressure fluctuation. *Int. J. Multiphase Flow* **10,** 711–720.

MATSUI, G. 1986 Automatic identification of flow regimes in vertical two-phase flow using differential pressure fluctuations. *Nucl. Engng Des.* **95,** 221–231.

MOON, F. 1987 *Chaotic Oscillations.* Wiley-Interscience, New York.

TUTU, N. K. 1982 Pressure fluctuations and flow pattern recognition in vertical two-phase gas-liquid flow. *Int. J. Multiphase Flow* **8,** 443–447.

TUTU, N. K. 1984 Pressure drop fluctuations and bubble–slug transition in a vertical two-phase water flow. *Int. J. Multiphase Flow* **10,** 211–216.

VINCE, M. A. & LAHEY, R. T. JR 1982 On the development of an objective flow regime indicator. *Int. J. Multiphase Flow* **8,** 93–124.